Black holes behave as thermodynamic objects

\[ T = \frac{\hbar \kappa}{2\pi c} \]

\[ S_{BH} = \frac{A}{4\hbar G} \]

Quantum (\(\hbar\)) and gravitational (\(G\))

Does this thermodynamic behavior have a microscopic explanation?
The problem of “universality”

Black hole entropy counts:

- Weakly coupled string and D-brane states
- Horizonless “fuzzball” geometries
- States in a dual conformal field theory “at infinity”
- Spin network states crossing the horizon
- Spin network states inside the horizon
- “Heavy” degrees of freedom in induced gravity
- Entanglement entropy (maybe holographic)
- Points in a causal set in the horizon’s domain of dependence
- States of a conformal field theory near an extremal horizon
- No local states—it’s inherently global
- Nothing—it comes from quantum field theory in a fixed background, and doesn’t know about quantum gravity

Answer: apparently, all of the above

Is there an underlying mechanism that can explain why these approaches all agree?
A small detour: entropy and the Cardy formula

Any two-dimensional conformal field theory can be characterized by generators $L[\xi], \bar{L}[\bar{\xi}]$ of holomorphic and antiholomorphic diffeomorphisms

Virasoro algebra:

$$[L[\xi], L[\eta]] = L[\eta \xi' - \xi \eta'] + \frac{c}{48\pi} \int dz \left( \eta' \xi'' - \xi' \eta'' \right)$$

Central charge $c$ (“conformal anomaly”) depends on theory
Conserved charge $L_0 \sim$ energy

Cardy: the density of states at temperature $T$ is asymptotically

$$\ln \rho(L_0) \sim \frac{\pi^2}{3} cT$$

Entropy is fixed by symmetry, independent of details!
Why this might help:

matter near a horizon looks conformal

Black hole in “tortoise” coordinates:

\[ ds^2 = N^2 (dt^2 - dr_*^2) + ds_\perp^2 \]

\( (N \to 0 \text{ at horizon}) \)

Scalar field:

\[ (\Box - m^2)\varphi = \frac{1}{N^2} (\partial_t^2 - \partial_{r_*}^2)\varphi + O(1) \]

Mass and transverse excitations become negligible
Effective two-dimensional conformal field (at each point)

Wilczek, Robinson, Iso, Morita, Umetsu:

two-dimensional CFT gives Hawking flux, spectrum

Medved, Martin, Visser:

conformal symmetry is generic at Killing horizon
An ADM Approach

Basic philosophy:
– Conditional probability: must impose presence of a black hole
– Need “boundary conditions” at horizon
– Diffeomorphisms must respect boundary conditions:
  some generators become symmetries rather than invariances
– New Goldstone-like degrees of freedom

More specifically:
– Conformal anomaly breaks diffeomorphism invariance

\[ L[\xi]|\text{phys}\rangle = \bar{L}[\xi]|\text{phys}\rangle = 0 \]

Not consistent with Virasoro algebra with \( c \neq 0 \):

Must weaken constraints— e.g., only require positive-frequency part annihilate \( |\text{phys}\rangle \)
\( \Rightarrow \) formerly nonphysical “gauge” states become physical
Strategy:

Brown and Henneaux: look at boundary term in ADM Hamiltonian

\[ \delta H[\xi] = \text{bulk terms} + \int_{\partial \Sigma} \Theta[\delta g] \]

\[ \delta B[\xi] = -\int_{\partial \Sigma} \Theta[\delta g] \]

Full Hamiltonian \((H + B)[\xi]\) generates diffeos/surface deformations, so have Dirac brackets

\[ \{(H + B)[\xi], (H + B)[\eta]\} = \delta_\xi (H + B)[\eta] \approx \delta_\xi B[\eta] \]

\[ \Rightarrow \{B[\xi], B[\eta]\}^* = -\int_{\partial \Sigma} \Theta[\delta g] \]

General structure:

\[ \{B[\xi], B[\eta]\}^* = B[\{\xi, \eta\}] + K[\xi, \eta] \]
Implementation:

Metric

\[ ds^2 = -N^2 dt^2 + d\rho^2 + \sigma_{\alpha\beta} dx^\alpha dx^\beta \]

Surface gravity

\[ \kappa = n^a \partial_a N \]

Stretched horizon \( N = \epsilon \); restrict diffeomorphisms to

\[ \delta_\xi N = 0 \Rightarrow \xi^\rho = -\rho \partial_t \xi^t = -\frac{1}{\kappa} \partial_t \xi^\perp \]

\[ \delta g_{\rho\alpha} = 0 \Rightarrow \partial_\rho \xi^\alpha = -\sigma^{\alpha\beta} \partial_\beta \xi^\rho \]
At stretched horizon, ADM Hamiltonian has a boundary term:

\[
\delta H[\xi] = \cdots - \frac{1}{16\pi G} \int_{\partial \Sigma} \left[ \sqrt{\sigma} \left( n^c g^{bd} - n^b g^{cd} \right) \left( \xi \perp \nabla_b \delta g_{cd} - \nabla_b \xi \perp \delta g_{cd} \right) + 2\xi^a \delta \pi_a^\rho - \xi^\rho \pi^{ab} \delta g_{ab} \right]
\]

Evaluate for a surface deformation \( \eta \):

\[
\delta_\eta H[\xi] = \cdots - \frac{1}{8\pi G} \int_{\partial \Sigma} \left[ \xi \perp \sigma^{ab} \nabla_a \nabla_b (n_c \eta^c) + n^a \partial_a \xi \perp \sigma^{bc} \nabla_b \eta_c - (\xi \leftrightarrow \eta) \right] \sqrt{\sigma}
\]

\[
= \cdots - \frac{1}{16\pi G \kappa} \int_{\partial \Sigma} \left[ \partial_t \xi \perp \Delta \Sigma \eta \perp - \partial_t \eta \perp \Delta \Sigma \xi \perp \right] \sqrt{\sigma}
\]
Need one more relation between $\partial_\alpha$ and $\partial_t$:
possibly from existence of Hamiltonian

$$\Delta_\Sigma \xi \perp - \frac{1}{N^2} \partial_t^2 \xi \perp = 0$$

Then have central term

$$\frac{1}{16\pi G \kappa} \int_{\partial \Sigma} \left[ \partial_t \xi^t \partial_t^2 \eta^t - \partial_t \eta^t \partial_t^2 \xi^t \right] \sqrt{\sigma} \Rightarrow c = \frac{3A}{2\pi G \kappa}$$

Cardy formula then gives

$$S = \frac{\pi^2}{3} c T = \frac{\pi^2}{3} \cdot \frac{3A}{2\pi G \kappa} \cdot \frac{\kappa}{2\pi} = \frac{A}{4G}$$
Related approaches:

– Particular conformal algebras for BTZ black hole, extremal Kerr black hole
– Explicit construction of boundary dynamics in 2+1 dimensions
– Covariant phase space methods
– Near-horizon conformal symmetries
– Adding horizon condition as genuine constraint

Universality?

Does this symmetry show up in other ("unrelated") approaches?

– AdS/CFT: related to near-horizon CFT
– Loop quantum gravity: interesting coincidences in central charges...
– Fuzzballs: under investigation
– Path integral? (Cardy formula as measure?)